

UNIT 1: INTEGERS

Positive and negative numbers

- The number line*
- Absolute value of an integer*

Operating with integers

- Adding integers*
- Subtracting integers*
- Multiplying integers*
- Divided integers*
- Combined operations*

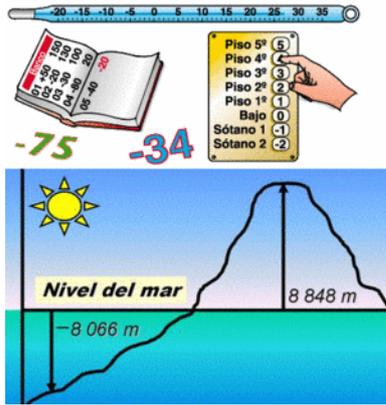
POSITIVE AND NEGATIVE NUMBERS

The set of **Natural Numbers** 1, 2, 3, 4, 5, ... is useful for solving many mathematical problems. However, there are situations where these numbers are not enough.

Sometimes we will need to use numbers below zero. Let's see some of them.

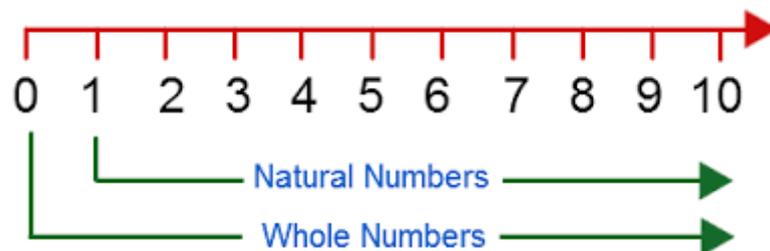
Examples:

- The temperature ("3 degrees below freezing")
- Different floors in a building ("2 floors below ground level")
- Money you have or you owe in a bank ("owing 30 €")
- Height and depth ("200 metres below sea level")



Integers are three types of numbers:

- **Positive integers:**
they are all the whole numbers greater than zero
- **Negative integers:**
they are all the opposites of these whole numbers
- **Zero**



- Positive numbers can be written with a positive sign "+" before the number, but we normally see them with no sign at all, and we assume the number is positive.

- Negative numbers are written with a negative sign "-" before the number

Examples:

"20 metres above sea level" would be written as +20 or just 20

"20 metres below sea level" would be written as -20

"I have 100 €" would be written as +100 or just 100

"I owe 100 €" would be written as -100

"2 floors above ground level" would be written as +2 or just 2

"2 floors below ground level" would be written as -2

In each case, a measurement is being taken from a reference position of zero.

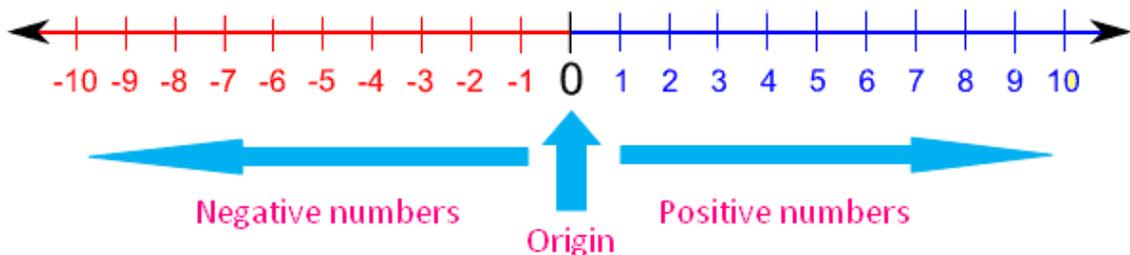
Let's see the most common uses of positive and negative signs:

Positive	Negative
Above	Below
Right	Left
Win	Lose
Increase	Decrease
Fast	Slow
North	South
East	West

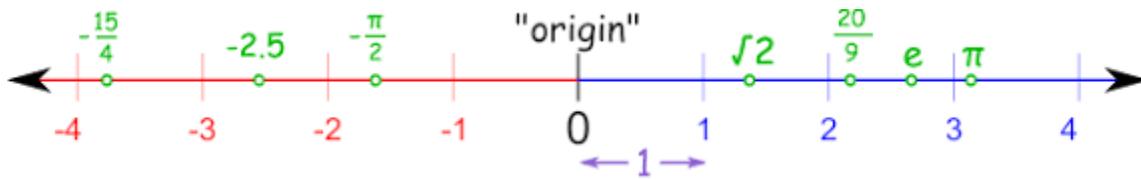
THE NUMBER LINE

The "**number line**" is a line labelled with the integers in increasing order, from left to right that extends in both directions.

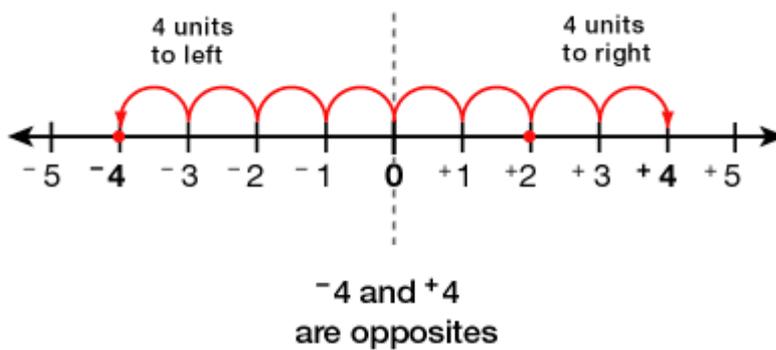
To draw this number line, first we place the number zero. Then we mark the positive numbers in equal intervals to the right. To set the negative numbers we can make a mirror image of the numbers to the right of zero.



The numbers to the right of zero are the positive numbers and the numbers to the left of zero are the negative numbers.



Pairs of numbers like $+4$ and -4 are exactly the same distance from 0, but on opposite sides, so they are called "**opposites**".



Using the position of numbers on a number line makes it easy to compare their size and arrange them in order.

As you move along the number line from left to right, the numbers increase in size. For any two different places on the number line, the integer on the right is greater than the integer on the left.

We use the following symbols to compare numbers:

**Greater
Than**

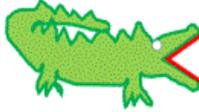


**Less
Than**



**Equal
To**



20  24
20 is less than 24

15  12
15 is greater than 12

Examples:

8 > 3 is read: "eight is greater than three"

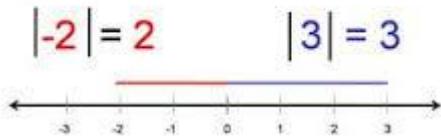
-4 < 0 is read: "negative four is less than zero"

ABSOLUTE VALUE OF AN INTEGER

The "**absolute value**"

Absolute Value
Definition

Absolute value is the distance from 0. The absolute value, just like distance, is always positive.



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of any number is the distance between that number and zero on the number line. It is always a positive number or zero.

We specify the absolute value of a number "a" writing "a" between two vertical bars.

- If the number is positive: the absolute value is the same number
- If the number is negative: the absolute value is the opposite

Examples:

$|+6| = 6$ $|-8| = 8$

$|x| = a$

- $x = +a$
- $x = -a$

OPERATING WITH INTEGERS

A number line is very useful for adding and subtracting positive and negative numbers.

ADDING INTEGERS

In order to add positive and negative integers, we will imagine that we are moving along that number line.

- **When we add a positive number to another**, we start at the first number mentioned and then move to the right the amount added.

Positive numbers make us move to the right side of the number line.

Examples:

$4 + 3 \rightarrow$ We start at 4 and go 3 units to the right. We end up at 7, so: $4 + 3 = 7$

$-2 + 6 \rightarrow$ We start at -2 and go 6 units to the right. We end up at 4, so: $-2 + 6 = 4$

- **When we add a negative number to another**, we start at the first number mentioned and then move to the left the amount added.

Negative numbers make us move to the left side of the number line.

Adding a negative number is the same as subtracting a positive number.

Examples:

$5 + (-2)$ is the same as $5 - 2 \rightarrow$ We start at 5 and go 2 units to the left, so: $5 + (-2) = 3$

$-4 + (-1)$ is the same as $-4 - 1 \rightarrow$ We start at -4 and go 1 unit to the left, so: $-4 + (-1) = -5$

Adding Integers

$$\oplus + \oplus = \oplus$$

$$\ominus + \ominus = \ominus$$

$$\oplus + \ominus = \oplus$$

$$\oplus + \ominus = \ominus$$

- When adding integers with the same sign: We add their absolute values, and give the result the same sign.
- When adding integers with the opposite signs: We take their absolute values, subtract the smallest from the largest, and give the result the sign of the integer with the larger absolute value.

Examples:

$$9 + 2 = 11 \quad -7 + 5 = -2 \quad -1 + 6 = 5 \quad -7 - 3 = -10$$

SUBTRACTING INTEGERS

In order to subtract positive and negative integers, we will imagine again that we are moving along that number line.

- **When we subtract a positive number from another,** we start at the first number mentioned and move the required number of units to the left.

So, subtraction is the opposite of addition.

Examples:

$4 - 7 \rightarrow$ We start at 4 and move 7 units to the left. We end up at -3, so: $4 - 7 = -3$

$-6 - 1 \rightarrow$ We start at -6 and move 1 unit to the left. We end up at -7, so: $-6 - 1 = -7$



- **When we subtract a negative number from another**, we start at the first number mentioned and move the required number of units to the right.

So, subtracting a negative number is the same as adding a positive number.

Examples:

$4 - (-2) \rightarrow$ We start at 4 and move 2 units to the right. We end up at 6, so: $4 - (-2) = 6$

$-5 - (-1) \rightarrow$ We start at -5 and move 1 unit to the right. We end up at -4, so: $-5 - (-1) = -4$

Subtraction rule:

- Subtracting an integer is the same as adding the opposite.

SUBTRACTION EQUALS ADDING THE OPPOSITE

$\oplus - \oplus$ changes to $\oplus + \ominus$

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$\ominus - \oplus$ changes to $\ominus + \ominus$

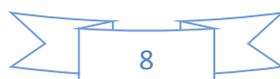
$\ominus - \ominus$ changes to $\ominus + \oplus$

The result of subtracting two integers could be positive or negative.

MULTIPLYING INTEGERS

In this section we will look for rules for the multiplication of positive and negative numbers.

- **The result of multiplying two numbers of the same sign is always a positive number.**
 - A positive times a positive gives a positive.
 - A negative times a negative gives a positive.



- **The result of multiplying two numbers of opposite signs is always a negative number.**
 - A positive times a negative gives a negative.
 - A negative times a positive gives a negative.
- **If one or both of the integers is 0, the product is 0.**
-

Look at the following chart:

Positive x Positive = POSITIVE answer	} If BOTH integers have SAME SIGN...answer is POSITIVE
Negative x Negative = POSITIVE answer	
Positive x Negative = NEGATIVE answer	} If the integers have DIFFERENT SIGNS...answer is NEGATIVE
Negative x Positive = NEGATIVE answer	

Examples:

$$4 \cdot 3 = 12 \quad -5 \cdot 3 = 15 \quad 7 \cdot (-3) = -21 \quad (-8) \cdot (-2) = 16$$

We have studied how to multiply two integers. When we want to multiply any number of integers, we follow these steps:

- We multiply their absolute values.
- We count the number of negative numbers in the products.
 - If the number of negative integers counted is even, then the product will be positive.
 - If the number of negative integers counted is odd, we will give the product a negative sign.

If any of the integers in the product is 0, the product is 0.

DIVIDING INTEGERS

In this section we will look for rules for the division of positive and negative numbers.

The rules for division are identical to those for multiplication. Remember that multiplication and division are **inverse operations**.

- **The result of dividing two numbers of the same sign is always a positive number.**

We divide the absolute value of the first integer by the absolute value of the second integer, and the result is positive.

- **The result of dividing two numbers of opposite signs is always a negative number.**

We divide the absolute value of the first integer by the absolute value of the second integer, and give this result a negative sign.

Dividing Integers Rules

+	÷	+	=	+
-	÷	-	=	+
+	÷	-	=	-
-	÷	+	=	-

Same Sign = Positive. Different Sign = Negative.

Look at the following chart:

Positive x Positive = POSITIVE answer	}	If BOTH integers have SAME SIGN...answer is POSITIVE
Negative x Negative = POSITIVE answer		
Positive x Negative = NEGATIVE answer	}	If the integers have DIFFERENT SIGNS...answer is NEGATIVE
Negative x Positive = NEGATIVE answer		

Examples:

$$10 : 5 = 2 \quad -12 : 6 = -2 \quad 15 : (-3) = -5 \quad (-8) : (-2) = 4$$

COMBINED OPERATIONS

We have studied the four operations: addition, subtraction, multiplication and division.

Now we will see the order we have to follow when we have to do combined operations.

The order of operations is:

1. **B**rackets are evaluated first.
2. **E**xponents are calculated next (we will study them in the next unit).
3. **D**ivisions and **M**ultiplications are done next, in the order that they appear.
4. **A**dditions and **S**ubtractions are then done, in the order that they appear.

The word **BEDMAS** may help you to remember this order.