Mathematics 2ESO
Unit 4

ALGEBRAIC EXPRESSIONS

REMARKABLE IDENTITIES

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Vocabulary and expressions

Write down on this page the new vocabulary and expressions we'll learn in class throughout this unit.

Common factors

English

We have seen how numbers can be expressed as the product of factors.

We know that prime numbers have only two different factors, the number itself and 1.

Factors that are prime numbers are called prime factors. Prime factors of any number can be found by repeated division.

For example:

$$30 = 2 \cdot 3 \cdot 5; \quad 45 = 3 \cdot 3 \cdot 5\ ; \ 120 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$$

Common factors and HCF (Highest common factor)

Notice that 3 and 5 are factors of both 30 and 45. They are called common factors. 15, which is $3 \cdot 5$, must also be a common factor.

Common factors are numbers that are factors of two or more numbers.

The **highest common factor** or **HCF** of the several numbers is the largest factor that is common to all of them.

To find the highest common factor of a group of numbers it is often best to express the numbers as products of primer factors. The common prime factors are identified and multiplied to give the HCF.

Algebraic products are also made up of factors. We can find the highest common factor of a group of algebraic products.

Factorisation is the process of writing an expression as a product of its factors.

Factorisation is the reverse process of expansion.

When we expand an expression we remove its brackets.

When we factorise an expression we insert brackets.

Notice that 5(x-1) is the product of the two factors, 5 and (x-1).

The brackets are essential since 5(x-1) multiplies 5 by the whole of x-1, whereas in 5x-1 only the x is multiplied by 5.

in **expansion**

$$5(x-1) = 5x - 5$$

is **factorisation**

To **factorise** an algebraic expression involving a number of terms, we look for the HCF of the terms. We write it in front of a set of brackets. We then identify the contents of the brackets.

For example, $6a^2$ and 2ab have HCF of 2a. So

$$6a^2 + 2ab =$$
$$= 2a \cdot 3a + 2a \cdot b =$$
$$= 2a\,(3a + b)$$

Factorising fully

Consider the expression 8x + 4 = 2 (4x + 2). This expression is not fully factorised since (4x + 2) still has a common factor of 2 which could be removed. So, although 2 was a common factor, it was not the HCF. The HCF is 4 and so

8x + 4 = 4 (2x + 1) is **fully factorised**.

Important: At this level of factorisation we are using the **distributive law** (*propiedad distributiva*), that says:

$$\mathbf{a \cdot (b + c) = a \cdot b + a \cdot c}$$

Factores comunes

Spanish

in **expansion**

$$5(x-1) = 5x - 5$$

is **factorisation**

1) Find the HCF of:

a) 18 y 27.

b) 50 y 75.

c) 25 y 15.

d) 98 y 42.

e) 32 y 36.

f) 48 y 84.

2) Find the unknown factor.

a) $4 \cdot \square = 8x$

b) $5 \cdot \square = 15y$

c) $3 \cdot \square = 9a^2$

d) $3x^2 \cdot \square = 12x^2$

e) $\square \cdot 7y = 7y^2$

f) $\square \cdot 3a = 6a^3$

g) $\square \cdot 2a = -8a$

h) $p \cdot \square = -pq$

i) $8s \cdot \square = -24st$

3) Find the HCF of:

a) $4x$ and 8

b) $3a$ and a

c) $5b$ and 15

d) $7a$ and $5a$

e) $7x$ and 9

f) $8c$ and $-24c$

g) 18b and 27b

h) 6d and - 15d

i) $5t^2$ and 25t

j) $9a^2b$ and $18ab^2$

k) 6abc, 8ab and 12bc

l) $16x^2z$ and $24xz^2$

Difference of two squares. Factorization

English

On expanding, $(a+b)(a-b) = a^2 - ab + ab - b^2 = a^2 - b^2$

So the factorization of the difference of squares is: $\mathbf{a^2 - b^2 = (a+b)(a-b)}$

Diferencia de cuadrados. Factorización

Spanish

4) Fully factorise:

a) $c^2 - d^2 =$

b) $m^2 - n^2 =$

c) $x^2 - 16 =$

d) $a^2 - 25 =$

e) $4x^2 - 1 =$

f) $4b^2 - 25 =$

g) $9y^2 - 16 =$

h) $49 - c^2 =$

i) $9 - 4y^2 =$

j) $x^4 - x^2 =$

k) $49a^2 - b^2 =$

l) $y^2 - 36x^2 =$

m) $9x^2 - 25y^2 =$

n) $9a^2 - 16b^2 =$

o) $b^2 - 81c^2 =$

p) $b^2c^2 - 4 =$

q) $36x^2 - p^2q^2 =$

r) $16b^2 - 25b^2c^2 =$

5) Fully factorise:

a) $3x^2 - 12 =$

b) $2b^2 - 50 =$

c) $4x^2 - 25 =$

d) $900 - 9b^2 =$

e) $48 - 3b^2 =$

f) $\pi R^2 - \pi r^2 =$

g) $10 - 10x^2 =$

h) $p^3 - p^4 =$

i) $x^3 - x =$

Difference of two squares. Expansion

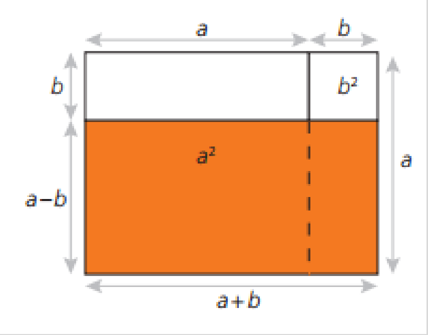
English

$(a+b)\,(a-b) =$

$= a^2 - ab + ab - b^2 =$

$= a^2 - b^2$

$$\mathbf{(a+b)\,(a-b) = a^2 - b^2}$$



Diferencia de dos cuadrados. Expansión

Spanish

6) Calculate the following products. Expand.

a) (x+1)·(x-1) =

b) (x+2)(x-2) =

c) (x+3)(x-3)=

d) (2+x)(2-x)=

e) (2x+1)(2x-1)=

f) (2x+5)(2x-5)=

g) (x+y)((x-y)=

h) (2x-y)(2x+y)=

i) (4x+3)(4x-3)=

j) (x2+1)(x2-1)=

k) (2x2+3)(2x2-3)=

l) (3-x2)(3+x2)=

m) (x2+y)(x2-y)=

n) (3x2+2y)(3x2-2y)=

Perfect square. Factorization

English

We have seen that

$(a+b)^2 =$
$= (a+b)(a+b) =$
$= a^2 + ab + ab + b^2 =$
$= a^2 + 2ab + b^2$

and

$(a-b)^2 =$
$= (a-b)(a-b) =$
$= a^2 - ab - ab + b^2 =$
$= a^2 - 2ab + b^2$

Expressions such as $\boldsymbol{a^2+2ab+b^2}$ and $\boldsymbol{a^2-2ab+b^2}$ are called **perfect squares** because they factorise into the product of two identical factors, or a factor squared.

$$\boldsymbol{a^2 + 2ab + b^2 = (a+b)^2} \quad \textbf{and} \quad \boldsymbol{a^2 - 2ab + b^2 = (a-b)^2}$$

For example, $x^2 + 6x + 9$ and $x^2 - 6x + 9$ are perfect squares because they factorise into two identical factors:

$$x^2 + 6x + 9 = (x+3)^2 \quad \text{and} \quad x^2 - 6x + 9 = (x-3)^2$$

You can check for yourself by expanding $(x+3)^2$ and $(x-3)^2$.

Identifying perfect squares

Notice that

$$(a+b)^2 = a^2 + 2ab + b^2 \quad \text{and} \quad (a-b)^2 = a^2 - 2ab + b^2.$$

perfect squares   perfect squares

So, a perfect square must be one of these two forms. It must contain two squares and a **middle term** of ±2ab.

For example

$x^2 + 10x + 25 =$
$= x^2 + 2·5·x + 5^2 =$
$= (x+5)^2 =$

and

$x^2 - 10x + 25 =$
$= x^2 - 2·5·x + 25 =$
$= (x-5)^2$

We can see that $x^2 + 10x + 25$ is a perfect square because $x^2 + 10x + 25$ contains two squares, x^2 and 5^2, and a middle term $2·5·x$.

$x^2 + 10x + 26$ does not satisfy these conditions.

Cuadrado perfecto. Factorización

Spanish

$$(a+b)^2 = a^2 + 2ab + b^2 \quad \text{and} \quad (a-b)^2 = a^2 - 2ab + b^2.$$

perfect squares   perfect squares

7) Find all the perfect squares of the form:

a) $x^2 + \boxed{\quad} + 1$

b) $x^2 + \boxed{\quad} + 4$

c) $x^2 + \boxed{\quad} + 16$

d) $4x^2 + \boxed{\quad} + 1$

e) $9x^2 + \boxed{\quad} + 4$

f) $16x^2 + \boxed{\quad} + 81$

g) $4x^2 + \boxed{\quad} + c^2$

h) $x^2 + \boxed{\quad} + 4d^2$

i) $a^2c^2 + \boxed{\quad} + 4$

8) Factorise:

a) $x^2 + 2x + 1 =$

b) $x^2 - 4x + 4 =$

c) $x^2 - 6x + 9 =$

d) $x^2 + 10x + 25 =$

e) $x^2 - 16x + 64 =$

f) $x^2 + 20x + 100 =$

g) $x^2 - 12x + 36 =$

h) $x^2 + 14x + 49 =$

i) $x^2 - 18x + 91 =$

j) $x^2 - 50x + 25 =$

9) Factorise:

a) $4x^2 + 4x + 1 =$

b) $16x^2 - 40x + 25 =$

c) $4x^2 + 28x + 49 =$

d) $4x^2 - 12x + 9 =$

e) $9x^2 + 6x + 1 =$

f) $9x^2 - 30x + 25 =$

10) Fully factorise:

a) $2x^2 + 4x + 2 =$

b) $2x^2 - 12x + 18 =$

c) $3x^2 + 30x + 75 =$

d) $-x^2 + 6x - 9 =$

e) $-x^2 - 8x - 16 =$

f) $-x^2 + 16x - 64 =$

g) $-2x^2 + 40x - 200 =$

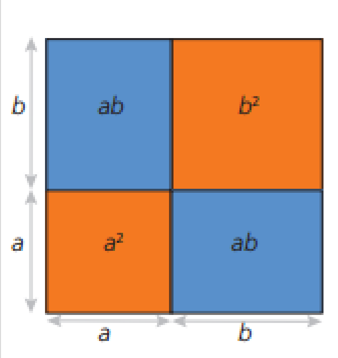
h) $-4b^2 + 28b - 49 =$

i) $ax^2 – 10ax + 25a =$

j) $27x2 – 18x + 3 =$

Perfect square. Expansion

English



Square of the sum

$(a+b)^2 =$

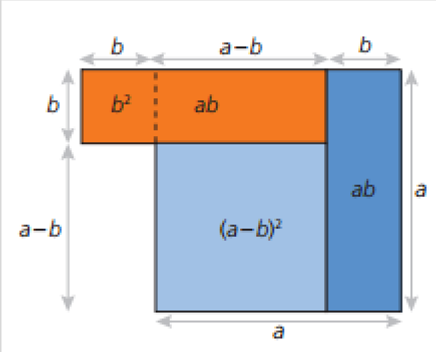
$= (a+b)(a+b) =$

$= a^2+ab+ab+b^2 =$

$= a^2+2ab+b^2$

$$\boldsymbol{(a+b)^2 = a^2 + 2ab + b^2}$$

Square of the difference



$(a-b)^2 =$

$= (a-b)(a-b) =$

$= a^2-ab-ab+b^2 =$

$= a^2-2ab+b^2$

$$\boldsymbol{(a+b)^2 = a^2 - 2ab + b^2}$$

Cuadrado perfecto. Expansión

Spanish

11) Expand the following perfect squares.

a) $(x+1)^2=$

b) $(x+2)^2=$

c) $(x+5)^2=$

d) $(2x+1)^2=$

e) $(2x+3)^2=$

f) $(3x+5)^2=$

g) $(x+y)^2=$

h) $(2x+y)^2=$

i) $(x-1)^2=$

j) $(x-2)^2=$

k) $(x-3)^2=$

l) $(2x-1)^2=$

m) $(2x-4)^2=$

n) $(1-x)^2=$

o) $(2-x)^2=$

p) $(5-2x)^2=$

q) $(2a-3)^2=$

r) $(a^2-b)^2=$

s) $(6+x)^2=$

t) $(3x^2+2y)^2=$

Factorizing quadratic trinomials

English

A **quadratic trinomial** is an algebraic expression of the form $ax^2 + bx + c$ where x is a variable and a, b, c are constants, $a \neq 0$.

Using FOIL, $(x+3)(x+6) = x^2 + \underbrace{6x + 3x}_{\text{the sum of the 'inners' and 'outers'}} + \underbrace{18}_{\text{the product of the 'lasts'}}$

So, $(x+3)(x+6) = x^2 + [6+3]x + [6 \times 3]$
$= x^2 + [\textbf{sum} \text{ of 6 and 3}]x + [\textbf{product} \text{ of 6 and 3}]$
$= x^2 + 9x + 18$

In order to factorise a quadratic trinomial such as $x^2 + 9x + 18$ into the form $(x + \ldots)(x + \ldots)$ we must find two numbers which have a *sum* of 9 and a *product* of 18.

In the general case, $\boldsymbol{x^2} + \underbrace{\boldsymbol{(a+b)x}}_{\text{the coefficient of } x \text{ is the sum of } a \text{ and } b} + \underbrace{\boldsymbol{ab}}_{\text{the constant term is the product of } a \text{ and } b} = \boldsymbol{(x+a)(x+b)}$

Spanish

Por ejemplo, Utilizando la propiedad distributiva:

$(x+3) \cdot (x+6) = x^2 + (6+3)x + (6 \cdot 3) =$
$= x^2 + (\text{suma de 6 y 3})x + (\text{producto de 6 y 3}) =$
$= x^2 + 9x + 18$

Para poder factorizar este trinomio cuadrático debemos encontrar dos números que sumen 9 y cuyo producto sea 18.

En general, $\mathbf{x^2 + (a+b)x + ab = (x+a) \cdot (x+b)}$

Un trinomio cuadrático es una expresión algebraica de la forma $ax^2 + bx + c$, donde x es la variable y a, b, c son constantes, con $a \neq 0$.

12) Find two numbers which have:

a) Product 8 and sum 6:

b) Product 21 and sum 10:

c) Product 14 and sum 9:

d) Product -6 and sum 5:

e) Product -7 and sum -6:

f) Product -22 and sum -9:

g) Product 16 and sum -10:

h) Product 24 and sum -11:

13) Factorizar:

a) $x^2 + 3x + 2 =$

b) $x^2 + 10x + 24 =$

c) $x^2 + 11x + 18 =$

d) $x^2 + 13x + 36 =$

e) $x^2 + 12x + 35 =$

f) $x^2 + 26x + 25 =$

g) $x^2 + 7x + 12 =$

h) $x^2 + 15x + 54 =$

i) $x^2 + 52x + 100 =$

j) $x^2 - 10x + 9 =$

k) $x^2 - 6x + 8 =$

l) $x^2 -13x + 12 =$

m) $x^2 -11x + 18 =$

n) $x^2 -14x + 33 =$

o) $x^2 - x - 2 =$

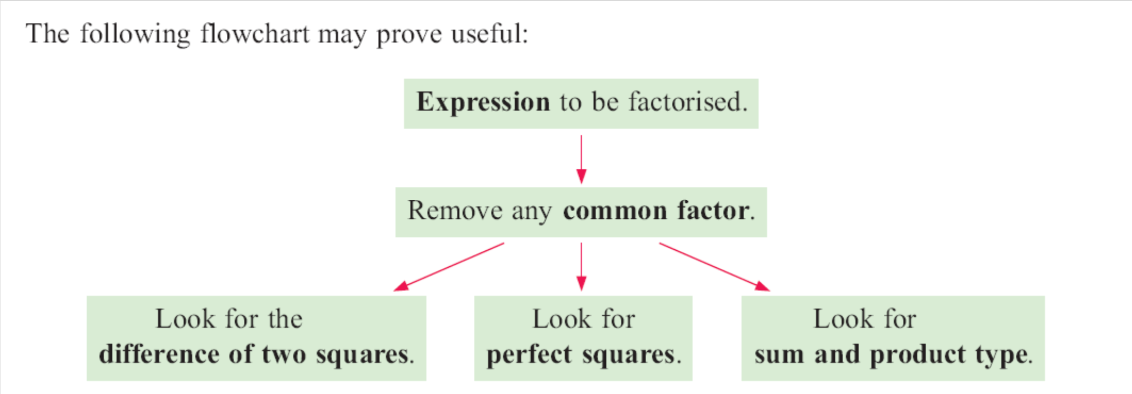
p) $x^2 - x - 6 =$

q) $x^2 + x - 6 =$

r) $x^2 - 2x +25 =$

Miscellaneous factorisation

English



Spanish

El siguiente diagrama es muy útil para factorizar expresiones algebraicas.

Expresión para ser factorizada

↓

Extraer factores comunes

↓

Diferencia de cuadrados | Cuadrados perfectos | Trinomio cuadrático

Por ejemplo, Utilizando la propiedad distributiva:

$(x+3) \cdot (x+6) = x^2 + (6+3)x + (6 \cdot 3) =$
$= x^2 +$ (suma de 6 y 3)x + (producto de 6 y 3) $=$
$= x^2 + 9x + 18$

Para poder factorizar este trinomio cuadrático debemos encontrar dos números que sumen 9 y cuyo producto sea 18.

En general, $\mathbf{x^2 + (a+b)x + ab = (x+a) \cdot (x+b)}$

14) Factorise:

a) $5a^2 + 10a =$

b) $6b^2 + 12 =$

c) $5x – 25y =$

d) $-x^2 – 16 =$

e) $q^2 + q^3 =$

f) $16x – 2x^3 =$

g) $y^2 – 8y + 15 =$

h) $6x^2 – 6x – 36 =$

i) $9c^2 – 81 =$

j) $x^2 – 8x + 16 =$

k) $x^3 – 16 x =$

15) Fully factorise, extracting first common factors:

a) $2x^2 + 10x + 12 =$

b) $2x^2 + 18x + 28 =$

c) $5x^2 - 10x - 15 =$

d) $6x^2 - 24x - 30 =$

e) $10x^2 - 80x + 120 =$

f) $x^3 - 9x^2 – 36x =$