Maths 2ESO

Unit 4

ALGEBRAIC EXPRESSIONS

IES Mata Jove - Course 2018/2019

Vocabulary and expressions

Write down on this page the new vocabulary and expressions we'll learn in class throughout this unit.

ACTIVITY 1

Guess the number [group work]

1. One of the members of the group reads card number 1
2. The rest of the group follows the instructions of the card using different numbers
3. Repeat with someone else reading
4. Repeat steps 1 to 3 with card 2
5. Try to reach a conclusion
6. Express what is written on the cards using algebraic language
7. Find out how they can guess the original number in the third card

1	Pick a number, add 2, multiply by 3, subtract 6, divide by 3. You get the number you started with.
2	Think of a number. Multiply by 5, add 8, multiply by 4, add 9, multiply by 5, subtract 105, divide by 100, subtract 1.
3	Pick a number, multiply by 5, add 6, multiply by 4, add 9, multiply by 5, tell me the number you got there. What else do you have to do to get to the number?

Albebraic notation

English

In algebra we use letters to represent variables.

We include the variables in expressions as though they were numbers.

However, we obey some rules which help make algebra easier.

In Algebra we agree:

- To leave out the "x" ("·") signs between any multiplied quantities provided that at least one of them is an unknown letter.
- To write numbers first in any product.
- Where products contain two or more letters, we write them in alphabetic order.

For example:

- 3b is used rather than 3 x b or b3.
- 3bc is used rather than 3cb.

Notación algebraica

Spanish

Writing sums as products

English

Sums of identical terms can be easily written using product notation. For example:

- $3 + 3 + 3 + 3 = 4 \cdot 3$ {4 lots of 3}
- $b + b + b + b = 4 \cdot b = 4b$ {4 lots of b}
- $a + a + a = 3a$
- $y + y + y + y + y = 5y$
- $r + r + r + s + s = 3r + 2s$
- $d + d - (a + a + a + a) = 2d - 4a$

Escribir sumas como productos

Spanish

Index notation

English

When the same number is multiplied by itself two or more times we use the index notation as a quick way of writing the product. For example:

- $3 \cdot 3 \cdot 3 \cdot 3 = 3^4$
- $b \cdot b \cdot b \cdot b = b^4$
- $8 \cdot b \cdot b \cdot a \cdot a \cdot a = 8a^2b^3$
- $k + k - 3 \cdot b \cdot b \cdot b = 2k - 3b^3$

Notación de potencias

Spanish

Simplifying algebraic products

English

$3 \cdot 2d$ and $a^2 \cdot 2ab$ are algebraic products.

Algebraic products can often be simplified using these steps:

- Expand out any brackets.
- Calculate the coefficient of the final product by multiplying all the numbers.
- Simplify the unknowns using index notation where appropriate. The unknowns should be written in alphabetic order.

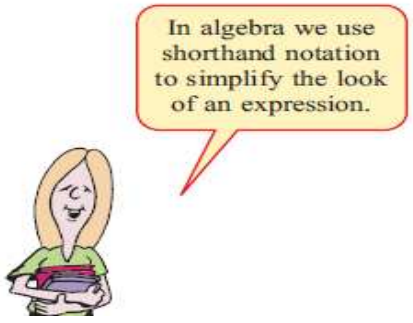
Simplificar productos algebraicos

Spanish

1) Simplify using product notation.

$3 \cdot m =$	
$n \cdot 7 =$	
$7 \cdot 3a =$	
$8m \cdot 3 =$	
$p \cdot 7 \cdot q =$	
$a \cdot b \cdot c =$	

$k \cdot 4 =$	
$6 \cdot 2b =$	
$m \cdot 5d =$	
$8m \cdot p =$	
$p \cdot q \cdot 2 =$	
$p \cdot h \cdot d =$	



2) Simplify using product notation.

$t + t =$	
$n + n + n =$	
$p + p + q + q + q + 3 =$	
$d + d + d + e =$	
$5 + y + y + x =$	
$a + a + a + a + b + b =$	

$a + a + 5 + a =$	
$b - (a + a) =$	
$b - a + a =$	
$t + t + t + s + s =$	
$5 + d + d + d =$	
$2 + a + b + b + a =$	

3) Simplify.

$a \cdot b + n =$	
$3 \cdot a + 2 \cdot b =$	
$b \cdot a + m =$	
$b \cdot a - c =$	
$d - a \cdot c =$	
$k - d \cdot 4 =$	

$c \cdot a + d \cdot b =$	
$10 - a \cdot b \cdot 2 =$	
$8 \cdot (m + n) =$	
$4 \cdot (d - 2) =$	
$(b - d) \cdot 3 =$	
$a \cdot b \cdot (c + 1) =$	

4) Explain why:

a) $3 - (a + a)$ is not the same as $3 - a + a$

b) $m - (n + n + n)$ ios not the same as $m - n + n + n$

5) Write as expanded form.

$a^3 =$	
$b^4 =$	
$3d^2 =$	
$4n^3 =$	
$10a^2b =$	

$4ab^3 =$	
$(2a)^2 =$	
$2a^2 =$	
$a^2 + 2b^2 =$	
$a^3 - 3b^2 =$	

6) Write in simplest form.

$3 \cdot a \cdot a =$	
$8 \cdot a \cdot a \cdot b =$	
$p \cdot p \cdot p \cdot p \cdot r \cdot r =$	
$a + a \cdot a =$	
$b \cdot b + 3 \cdot b =$	
$3 \cdot a \cdot b \cdot b - 5 \cdot b \cdot c =$	
$4 \cdot x + 2 \cdot x \cdot x \cdot x =$	
$2 \cdot a \cdot a \cdot 5 =$	
$3 \cdot p \cdot 4 \cdot q \cdot q =$	

$5 \cdot b \cdot b \cdot b =$	
$5 \cdot m \cdot n \cdot n \cdot n =$	
$4 \cdot a \cdot 5 \cdot a =$	
$a \cdot a \cdot a + a =$	
$c \cdot c \cdot c - 5 \cdot c =$	
$3 \cdot a \cdot a + a \cdot m =$	
$4 \cdot (x + 2 + x) \cdot x =$	
$3 \cdot d \cdot d \cdot 2 \cdot c =$	
$b \cdot b \cdot b - a \cdot a =$	

The language of Mathematics

English

Some key words used in Algebra are:

Word	*Meaning*	*Example(s)*
variable	an unknown value that is represented by a letter or symbol	$C = 2\pi r$ has variables C and r.
expression	an algebraic form consisting of numbers, variables and operation signs	$2x + y - 7$, $\dfrac{2a+b}{c}$
equation	an algebraic form which contains an $=$ sign	$3x + 8 = -1$, $\dfrac{x-1}{2} = -4$
terms	algebraic forms which are separated by $+$ or $-$ signs, the signs being included	$3x - 2y + xy - 7$ has four terms. These are $3x$, $-2y$, xy and -7.
like terms	terms with exactly the same variable form	In $4x + 3y + xy - 3x$: • $4x$ and $-3x$ are like terms • $4x$ and $3y$ are unlike terms • xy and $3y$ are unlike terms.
constant term	a term which does not contain a variable	In $3x - y^2 + 7 + x^3$, 7 is a constant term.
coefficient	the number factor of an algebraic term	In $4x + 2xy - y^3$: • 4 is the coefficient of x • 2 is the coefficient of xy • -1 is the coefficient of y^3.

El lenguaje de las Matemáticas

Spanish

		$\frac{2a + b}{c}$
		$\frac{x - 1}{2} = -4$

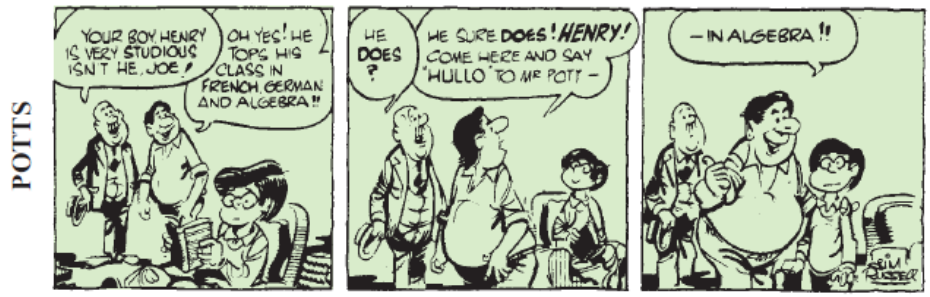
Changing words to symbols

English

In Algebra wwe can convert sentences into algebraic expressioins or equations. For example:

	Twice a number	increased by 7	is	5 less than the number
becomes	$2x$	$+\ 7$	$=$	$x-5$.

Many algebraic statements contain words such a sum, difference, product, and quotient.



Word	*Meaning*	*Example*
sum	The sum of two or more numbers is obtained by **adding** them.	$3+7$, $a+4$, $b+c+d$ are sums.
difference	The difference between two numbers is the larger one **minus** the smaller one.	$9-5$, $d-6$ (if $d>6$) are differences.
product	The product of two or more numbers is obtained by **multiplying** them.	3×6, $3a$, xyz are products.
quotient	The quotient of two numbers is the first one mentioned **divided** by the second.	The quotient of x and y is $\frac{x}{y}$.
average	The **average** of a set of numbers is their sum divided by the number of numbers.	The average of a, b and c is $\frac{a+b+c}{3}$.

Cambiando palabras por símbolos *Spanish*

$$\underbrace{\text{Twice a number}}_{2x} \quad \underbrace{\text{increased by } 7}_{+\,7} \quad \text{is} \quad \underbrace{\text{5 less than the number}}_{x-5.}$$

becomes

		$\frac{x}{y}$
		$\frac{a+b+c}{3}$

7) Write expressions for the:

sum of:		product of:		quotient of:		average of:	
7 and 3		7 and 3		7 and 3		7 and 3	
4 and y		4 and y		4 and y		4 and y	
t and 2p		t and 2p		t and 2p		t and 2p	
a, b and c		a, b and c		a+b and c		a,b and c	

8) Write expressions for the difference between:

7 and 3		4 and y if 4 < y	
4 and y if 4 > y		(a+b) and c if (a+b) <c	

9) Write down algebraic expressions for the sum of:

b and c divided by 3		B and c, all divided by 3	
2 and x squared		2 and x, all squared	

10) Write down algebraic expressions for:

one third of the sum of r and s	
the sum of m and n cubed	
triple the sum of b and c	
the sum of m and n, all cubed	
three times x, subtracted from b	
the sum of a and five times b	
4 more than a	
the square of the product of c and d	
the sum of the squares of p, q and r	
the product of a and the square of b	
3 less than t	
the product of the square of x and 9	
the sum of the squares of x and y	
half the sum of c and d squared	

11) Escribe las siguientes fórmulas usando el lenguaje algebraico, asignando una variable a cada magnitud.

La velocidad es el espacio recorrido dividido del tiempo empleado	
El área de un triángulo es el producto de la longitud de la base por la de la altura, dividido entre dos	
The diameter of a circumference is the double of the radius	
The perimeter of a square y four times the length of the side	

12) Write the following statements using algebraic language.

El cubo de un número menos la mitad de otro número	
The difference between two numbers is the double of other number	
La diferencia de un número y de su cuadrado	
The product of three numbers is 0	
La cuarta parte de un número más su mitad es 100	
The subtraction of two numbers is 3	
Un número más su siguiente es el cuadrado de dicho número	
The square of the sum of two numbers	

Generalising Arithmetic

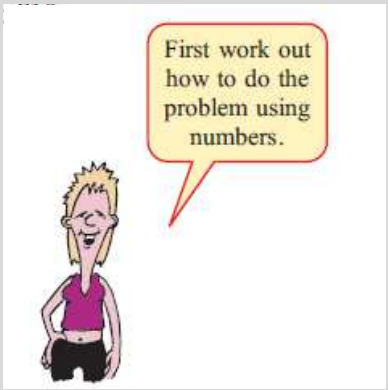
English

To find algebraic expression for many real world situations, we first think in terms of numbers or numerical cases. We then proceed to more general cases.

For example, suposse we are asked to find the total cost of x books which each cost y €. We could start by finding the total cost of 5 books which each cost 10 €.

In this case the total cost is 5 · 10 e = 50 €.

We multiplied the two quantities, and so the total cost of x books at y € each is x · y € = xy €.



Generalizando la Aritmética

Spanish

13) Find the total cost of buying:

5 toys at 300 € each.	
a toys at 300 e each.	
a toys at c € each.	
8 pears at 50 cents each.	
p pears at 50 cents each	
p pears at y cents each.	

14) Find the change from 100 e when buying:

3 boxes of chocolates at 11 € each.	
x boxes of chocolates at 11 € each..	
x boxes of chocolates at y € each	

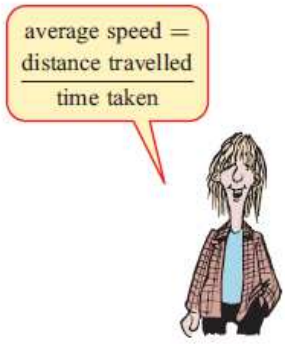
15) Albert is now 14 years old. How old was he x years ago?

16) Anne can paddle her canoe at 3 km per hour. How far can she paddle in t hours?

17) Katia is climbing the stairs to the top of a 35 m tower. Each step is 21 cm high. If Katia has climbed x steps:

a) How high has she climbed?

b) How much further must she climb to the top?



18) Graham buys c chairs and t tables. Find the total cost if each chair costs 45 € and each table costs 85 €.

19) Su Lin travelled a km in 3 hours and then b km in 2 hours. Find:

a) The total distance Su Lin travelled.

b) The average speed for the whole trip.

20) A lorry driver travels at an average speed of 80 km per hour for 5 hours.

a) How far has the driver travelled?

b) How far would the driver travel at an average of s km per hour for t hours?

21) Escribe en lenguaje algebraico indicando claramente qué significa cada variable.

Años de Ana dentro de 12 años; años de Ana hace 5 años; la mitad de la edad actual de Ana	
Años de Isabel hace 3 años; años de Isabel dentro de 7 años; el doble de la edad actual de Isabel	
La edad de una mujer es el doble de la de su hijo menos 5 años. La mujer tiene 3 años más que su marido	
Un padre tiene 22 años más que un hijo y 27 más que su hija	
María tiene la mitad del dinero de Raquel más 34€; Raquel tiene 2€ menos que Lucía.	
Ramón tiene el doble de hermanos que Luis y dos más que Pedro.	

22) Translate these algebraic expressions into English.

$2 = 3p - q$	
$x \cdot y = 4$	
$2n - 10 = n$	
$2x^2$	
$x - y : 2$	
$(x - y) : 2$	
$(x + y)(x - y)$	
$2(x^2 - y^2)$	
xy^2	
$(xy)^3$	
$a:b + ab$	
$(2a + b)^4$	

23) Escribe las siguientes fórmulas usando el lenguaje algebraico y asignando una variable a cada magnitud.

La fuerza es la masa por la aceleración	
El área de un rectángulo es el producto de los lados	
La densidad es la masa entre el volumen	
La velocidad es la aceleración por el tiempo	

24) Escribe usando lenguaje algebraico.

Hay 113 personas en un avión. Si x de esas personas se bajan, ¿cuántas personas quedan en el avión?	
Hay x personas en un avión y 113 de ellas se bajan. ¿Cuántas personas quedan en el avión?	
Un tren tiene 86 pasajeros. En una estación, suben p pasajeros y bajan q. ¿Cuántos pasajeros quedan en el tren?	
Un hotel tiene x pisos con y apartamentos por piso. Cada apartamento tiene 4 habitaciones. ¿Cuántas habitaciones hay en total en el hotel?	
Una lata de refresco cuesta c céntimos, una chocolatina 80 céntimos. ¿Cuántos euros costarán 7 latas y n chocolatinas?	

25) El precio de las entradas para un concierto es 40 € para adultos y 15 € para los niños. Si un grupo de x adultos e y niños van al concierto, escribe la expresión algebraica que da el coste total. Calcula también el coste total para un grupo de 5 adultos y 9 niños. (*Notebook*)

26) Alicia está en la frutería y tiene que comprar tomates. Si el precio es de 1,80 €/kg y representamos por c la cantidad de tomates comprados y p el precio que tendrá que pagar Alicia por ellos, encuentra la expresión algebraica que *relaciona* c con p. ¿Cuánto pagará Alicia por 1,2 kg de tomates?, ¿y por 820 gr? (Notebook)

27) En un gimnasio cobran 30 € por la inscripción y 45 € al mes. Si representamos por x la cantidad de meses que una persona asiste al gimnasio y por y el precio que paga, encuentra la expresión algebraica que relaciona x e y. ¿Cuánto cuesta el primer año de gimnasio? (*Notebook*)

28) La agencia inmobiliaria Tucasa determina el sueldo de sus empleados del modo siguiente: un fijo mensual de 750 €, 100 € por cada piso alquilado y 420 € por cada piso vendido. Escribe la fórmula que permite calcular el sueldo mensual de un empleado. ¿Cuánto ganó Juan el mes pasado si alquiló 3 pisos y vendió 1? (*Notebook*)

29) Tenemos un coche que consume 5 litros de gasolina cada 100 kilómetros recorridos. Escribe la expresión algebraica que da los litros consumidos en x km. (Notebook)

30) Tres mangueras iguales tardan 25 minutos en llenar una piscina hinchable. ¿Cuántas mangueras son necesarias para llenar la piscina en x minutos? (*Notebook*)

Algebraic expressions. Clasification

English

Monomials

A **monomial** is an *algebraic expression* where the only operations between the variables are the product and power of the natural exponent.

$$2x^2y^3z$$

Elements of a Monomial

- ***Coefficient***

  The **coefficient** of **a monomial** is the number that multiplies the variable(s).

- ***Literal Part***

  The **literal part** is formed by the variables (letters) and its exponents.

- ***Degree***

  The **degree** of a **monomial** is the sum of all exponents of the letters or variables.

  The degree of $2x^2 y^3 z$ is: 2 + 3 + 1 = 6

- **Similar Monomials**

  Two monomials are similar when they have the same literal part.

  $2x^2y^3z$ is similar to $5x^2y^3z$

- The **opposite** of a monomial has a different sign.

  The opposite of 5y is -5y.

Operaciones algebraicas. Clasificación

Spanish

Algebraic expressions. Operations

English

Adding Monomials

In order to add monomials, they have to be similar.

The sum of the monomials is another monomial whose literal part is the same and the coefficient is the sum of the coefficients.

$$\mathbf{ax^n + bx^n = (a + b)x^n}$$

$$2x^2\ y^3\ z + 3x^2\ y^3\ z = 5x^2\ y^3\ z$$

The sum of two or more non-similar monomials is a **polynomial**.

$$2x^2\ y^3 + 3x^2\ y^3\ z$$

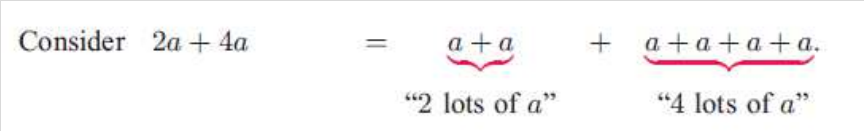
Collecting like terms

Like terms are algebraic terms which contain the same variables to the same indices.

For example:

- 2xy and -2xy are like terms
- a^2 and -3a are unlike terms because the indices of a are not the same

Algebraic expressions can often be simplified by adding or subtracting like terms. This is sometimes called **collecting like terms**.



In total we have 6 lots of a, and so 2a + 4a = 6a

Example: Simplify, where possible, by collecting like terms:

- 3x + 2x = 5x
- 7a – 3a = 4a
- -2x + 3 –x = -3x + 3
- 3bc + bc = 4 bc
- 2x – x^2 is in simplest form

Operaciones algebraicas. Operaciones

Spanish

Consider $2a + 4a = \underbrace{a + a}_{\text{“2 lots of } a\text{”}} + \underbrace{a + a + a + a}_{\text{“4 lots of } a\text{”}}.$

31) Simplify, where possible, by collecting like terms:

$3 + x + 5 =$	$8 + 7 + x =$
$p + 3 + 7 =$	$12 + a + a =$
$b + 3 + b =$	$b + b =$
$2x + x =$	$a + 3 + a + 7 =$
$x + 3x =$	$3x - 2x =$
$3x - x =$	$a^2 + a^2 =$
$7x + 3 =$	$2x^2 + x^2 =$
$17x - 7 =$	$17x - x =$
$3b^2 - b^2 =$	$2ab + 3ab =$
$g + g + g =$	$9b - 7b - 2 =$
$11n - 11n =$	$11n - n =$
$11n - 11 =$	$3ab + ba =$
$xy + 2xy =$	$2p^2 - p^2 =$
$3a + 2 + a + 4 =$	$2a + 3a + 4a =$
$b + 3 + 2b + 4 =$	$3xy + 4yx =$
$2a + b + 3a + b =$	$3a^2 + a + a^2 + 2a =$
$3x + 2x - x =$	$n + 2n - 3n =$
$ab + b^2 + 2ab + b^2 =$	$3x + 7x - 10 =$
$3x + 7x - 10x =$	$3x + 7x - x =$
$r + r + 2r^2 =$	$x^2 + x + 2 =$
$3 + 6y - 1 + 2y =$	$-2x + 3x =$

32) Simplify, by collecting like terms:

$3x + 8x =$	$3x - 8x =$
$-3x + 8x =$	$-3x - 8x =$
$5a + a =$	$5a – a =$
$-5a + a =$	$-5a – a =$
$m2 + 2m =$	$-3d - 5d =$
$-3d + 5d =$	$3d - 5d =$
$b + 2b – 3 =$	$t - 3t - 2t =$
$-6g – g =$	$4m - 7m + 1 =$
$a + 2 -3a =$	$-2b - (-3b) =$
$3b – b =$	$3b - (-b) =$
$x - (-2x) =$	$a + 2 - 2a – 5 =$
$ab + 2 - 3ab -4 =$	$3x – 4 + 4 - 4x =$
$a - 2b + 3a + b =$	$ab + 3ab - 4ab =$
$2x^2 – 7 + x^2 – 3 =$	$3a + b - 2a - 7b =$
$5bc - 8bc + 3 =$	$3x^2 + x -x^2 - 2x =$
$-x^3 - x^2 + x^3 - 3x^2 =$	$2x – y - (-x) –y =$
$-2x – 4 - 3x – 6 =$	$-(-x) + 2x =$

Algebraic expressions. Operations

English

Multiplying Monomials

The product of two or more monomials is another monomial whose coefficient is the product of the coefficients and whose literal part is product of the powers with the same base.

$$\mathbf{ax^n \cdot bx^m = (a \cdot b)(x^n \cdot x^m) = (a \cdot b)x^{n+m}}$$

$$(5x^2\ y^3\ z) \cdot (2\ y^2\ z^2) = 10\ x^2\ y^5\ z^3$$

For example:

$2x \cdot 5 = 10x$
$4x \cdot 3x^2 = 12\ x^3$
$6x^3 \cdot 5x^2 =$
$-2x \cdot 3x^2 = -6\ x^3$

Operaciones algebraicas. Operaciones

Spanish

33) Write the following algebraic expressions in simplest form:

$x \cdot y =$	$x \cdot 3 \cdot y =$
$(-x)^2 =$	$x^2 \cdot x =$
$x \cdot y \cdot z =$	$(-x) \cdot 2x =$
$a \cdot 2b =$	$(-a) \cdot a^2 =$
$x^2 \cdot x^2 =$	$x^3 \cdot x^2 =$
$x^4 \cdot x^3 =$	$x^3 \cdot x^6 =$

34) Simplify the following:

$3a \cdot b =$	$3a \cdot b^2 =$
$3ab \cdot 2b =$	$5ab \cdot 4ab =$
$(4a)^2 =$	$(3b)^2 \cdot b^2 =$
$4y \cdot y^2 =$	$5b^2 \cdot 2b =$
$5b^2 \cdot b^2 =$	$3b^2 \cdot 5b^3 =$
$4x \cdot (-x) =$	$(-3x) \cdot x =$
$2x \cdot (-3x) =$	$(-2x) \cdot (-4x) =$
$(-x^2) \cdot 2x =$	$3x^2 \cdot (-6x) =$
$5 \cdot (-x^3) =$	$2x \cdot (-x)^3 =$
$4d^2 \cdot (-d) =$	$(2x)^3 =$

Algebraic expressions. Operations

English

Dividing Monomials

Monomials can only be divided if they have the same literal part and the degree of the dividend is greater than or equal to the corresponding divisor.
The division of monomials is another monomial whose coefficient is the quotient of the coefficients and the literal part is the quotient of the powers with the same base.

$$\frac{6x^3y^4z^2}{3x^2y^2z^2} = 2xy^2$$

If the degree of the divisor is greater, an algebraic fraction is obtained.

$$\frac{6x^3y^4z^2}{3x^5y^2z^4} = \frac{2y^2}{x^2z^2}$$

QUOTIENTS

The **quotient** of two factors is found by dividing the first by the second. The result is an **algebraic fraction**.

When we divide algebraic expressions we can cancel common factors in exactly the same way as for numerical fractions.

Example 15 — Self Tutor

Simplify: a $\frac{6x^3}{3x}$ b $\frac{4x^2}{12x^4}$

a
$$\frac{6x^3}{3x} = \frac{\overset{2}{\cancel{6}} \times x \times x \times \overset{1}{\cancel{x}}}{\underset{1}{\cancel{3}} \times \underset{1}{\cancel{x}}} = \frac{2x^2}{1} = 2x^2$$

b
$$\frac{4x^2}{12x^4} = \frac{\overset{1}{\cancel{4}} \times \overset{1}{\cancel{x \times x}}}{\underset{3}{\cancel{12}} \times x \times x \times \underset{1}{\cancel{x \times x}}} = \frac{1}{3x^2}$$

Operaciones algebraicas. Operaciones

Spanish

35) Simplify the following:

$\frac{x^4}{x^2} =$	$\frac{x^4}{x} =$	$\frac{x^5}{x^2} =$
$\frac{x^5}{x^3} =$	$\frac{3x^3}{x} =$	$\frac{5x^4}{x^2} =$
$\frac{10x^3}{2x} =$	$\frac{10x^4}{5x} =$	$\frac{4x^4}{2} =$
$\frac{4x^4}{2x} =$	$\frac{4x^4}{2x^2} =$	$\frac{4x^4}{2x^3} =$
$\frac{4x^4}{2x^4} =$	$\frac{2x^4}{4x^3} =$	$\frac{5x^4}{10x} =$
$\frac{8x}{4x^2} =$	$\frac{4x}{8x^2} =$	$\frac{4x}{8x^3} =$
$\frac{3x^2}{6x} =$	$\frac{3x}{6x^2} =$	$\frac{3x^2}{6x^3} =$

Algebraic expressions. Operations *English*

Power of a Monomial

To calculate the power of a monomial, every element of the monomial is raised to the exponent of the power.

$$\mathbf{(ax^n)^m = a^m \cdot x^{n \cdot m}}$$

$(2x^3)^3 = 2^3 \cdot (x^3)^3 = 8x^9$
$(-3x^2)^3 = (-3)^3 \cdot (x^3)^2 = -27x^6$

Operaciones algebraicas. Operaciones *Spanish*

36) Calculate the power of the following:

$(-x)^2 =$	$(-x)^3 =$	$(2x)^2 =$	$(2x)^3 =$
$(-2x)^2 =$	$(-2x)^3 =$	$(3x)^2 =$	$(3x)^3 =$
$(-3x)^2 =$	$(-3x)^3 =$	$(-x^2)^2 =$	$(-x^2)^3 =$
$(2x^3)^2 =$	$(2x^2)^3 =$	$(-2x^2)^2 =$	$(-2x^3)^3 =$

Algebraic expressions. Clasification

English

A **polynomial** is a monomial or the sum or difference of non similar monomials.

Each monomial is called a **term** of the polynomial, and the term that hasn't literal part is called **constant term** (*término independiente*).

Important!: Terms are separated by addition signs and subtraction signs, but never by multiplication signs.

A polynomial with one term is called a monomial

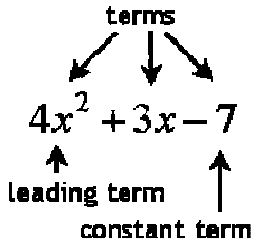
A polynomial with two terms is called a binomial

A polynomial with three terms is called a trinomial

Example*:*

Notice the exponents on the terms. The first term has an exponent of 2; the second term has an "understood" exponent of 1; and the last term doesn't have any variable at all. Polynomials are usually written this way, with the ***terms written in "decreasing" order***; that is, with the largest exponent first, the next highest next, and so forth, until you get down to the plain old number.

The exponent on a term tells you the "degree" of the term. For instance, the leading term in the above polynomial is a "second-degree term" or "a term of degree two". The second term is a "first degree" term. The degree of the leading term tells you the **degree** of the whole **polynomial**; the polynomial above is a "second-degree polynomial".



The **opposite** of a polynomial P(x) is obtained changing the sign of all the terms of the polynomial. It's written –P(x).

Operaciones algebraicas. Clasificación

Spanish

37) Escribe las siguientes expresiones algebraicas de la forma más simple posible. Clasifica las expresiones obtenidas en monomio o binomio. Indica coeficientes, variables y grado.

	Simplest form	Monomial or binomial	Coeficient	Variables	Degree
$d \times c$					
$a \times b \times c$					
$s \times 5$					
$m \times 2 \cdot n$					
$3 \times t + 2$					
$n \cdot 7 - 4$					
$2 \cdot a \cdot 5 \cdot b$					
$3 \times x \times y$					
$3 \cdot x \cdot 7$					
$5 + s \cdot 3$					
$6 - t \times p$					
$m + 2 \cdot n$					
$2 + a \cdot 3 \times c$					
$(-1) \cdot x \cdot 3 + y$					
$a \cdot (-2) + 4 \cdot c \cdot b$					

38) Simplify the following algebraic expression by combining like terms. (*Notebook*)

Simplifica las siguientes expresiones algebraicas agrupando términos semejantes.

a) $2x+3x$

b) $3a+4b-5a+b$

c) $2+3x-1-x$

d) $3x^3-5x^2+x^3-2x^2$

e) $2ab-4ba+9ab$

f) $3x^2y+2xy^2+5yx^2$

g)

39) Simplify the following algebraic expressions by combining like terms. Classify the expressions obtained as monomials, binomials or trinomials. Write coefficients, variables and degree in each case. (*Notebook*)

a) $4p-2p+3p^2$

b) $-4-n+7n^2+3$

c) $-4s+3-2-s$

d) $-a^2+2a^2-3+b^2+5$

40) Una familia dedica 1/3 de sus ingresos mensuales al alquiler de la vivienda y 2/5 a alimentación. Si sus ingresos mensuales son de a euros, escribe la expresión algebraica que representa el dinero dedicado mensualmente a alquiler y alimentación. Calcula el gasto en cada apartado para un sueldo de 1800 €/mes. (*Notebook*)

Algebraic expressions. Operations

English

The distributive law

Example:

Jasmin earned 11 · 5€ form the mother and 11 · 7€ from her grandfather over the 11 week period.

Another way of looking at this is that Jasmin earned 11 lots of 5€ + 7€, which is 11·(5+7) €.

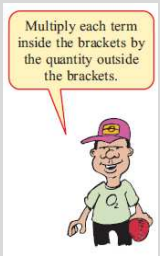
Consequently, 11(5+7) = 11·5+11·7.

Notice that the factor outside the brackets is multiplied by each term of inside the brackets.

a(b+c)=ab+ac is called the **distributive law**.

- The process of removing the brackets in a product is known as **expansion**:

From a(b+c) to ab+ac



- The process of transforming the sum into a product with brackets is known as **factorization**:

From ab+ac to a(b+c)

Example 1 — Self Tutor

Expand and simplify:

a $5(x+4)$ b $4(y-3)$

a $5(x+4)$
$= 5\times x + 5\times 4$
$= 5x+20$

b $4(y-3)$
$= 4(y+-3)$
$= 4\times y + 4\times(-3)$
$= 4y-12$

Example 2 — Self Tutor

Expand and simplify:

a $3(2a+7)$ b $2(3x-4)$

a $3(2a+7)$
$= 3\times 2a + 3\times 7$
$= 6a+21$

b $2(3x-4)$
$= 2\times 3x + 2\times(-4)$
$= 6x-8$

Operaciones algebraicas. Operaciones *Spanish*

41) Expand and simplify:

2(x+7)=	3(x-2)=	4(a+3)=
5(a+c)=	6(b-3)=	7(m+4)=
2(n-p)=	4(p-q)=	3(5+x)=
5(y-x)=	8(t-8)=	4(7+m)=
6(d+e)=	2(x-11)=	3(7+k)=
5(p-q)=	4(10-j)=	7(y+n)=
2(n-12)=	8(11-d)=	9(2x+1)=
3(1-2x)=	5(2a+3)=	11(1-2n)=
6(3x+y)=	5(x-2y)=	4(3b+c)=
2(a-2b)=	7(a-5b)=	12(2+3d)=
8(3-4y)=	6(5b+3ª)=	11(2x-y)=
4(p+9q)=	5(a-8b)=	2(9+8x)=
3(9x+y)=	7(c-9d)=	6(m+7n)=

Example 3 — Self Tutor

Expand and simplify:

a $2y(3y+5)$ b $2x(3-2x)$

a $2y(3y+5)$
$= 2y \times 3y + 2y \times 5$
$= 6y^2 + 10y$

b $2x(3-2x)$
$= 2x \times 3 + 2x \times (-2x)$
$= 6x - 4x^2$

42) Expand and simplify:

x(x+2)=	x(5-x)=	a(2a+4)=
b(5-3b)=	a(b+2c)=	$a(a^2+1)$=
a^2(2-a)=	2x(3-4x)=	3x(6-x)=
5x(x-4)=	4a(1-a)=	7b(b+2)=
(2x+3)x=	(5-2x)x=	ab(a+b)=
a^2b(3-b)=	mn(m-n)=	ac(c-4a)=
6p(4-7pq)=	$(3k+5t^2)$k=	$(7a^2-5b)$b=
Xy(x+9y)=	(7-4x)xy=	$(3t-5p^2)$t=

Example 4 — Self Tutor

Expand and simplify:

a $-4(x+3)$ b $-3(2x-4)$ c $-(3-2x)$

a $-4(x+3)$
$= -4 \times x + -4 \times 3$ {-4 is multiplied by x and by 3}
$= -4x - 12$

b $-3(2x-4)$
$= -3 \times 2x + -3 \times (-4)$ {-3 is multiplied by $2x$ and by -4}
$= -6x + 12$

c $-(3-2x)$
$= -1(3-2x)$ {-1 is multiplied by 3 and by $-2x$}
$= -1 \times 3 + -1 \times (-2x)$
$= -3 + 2x$

43) Expand and simplify:

-2(x+2)=	-3(x+4)=	-4(x-2)=
-5(5-x)=	-(a+2)=	-(x-3)=
-(5-x)=	-(2x+1)=	-3(4-x)=
-4(5x-2)=	-5(3-4c)=	-(x-2)=

Example 5 — Self Tutor

Expand and simplify:

a $-a(a+7)$ b $-4b(2b-3)$

a
$$-a(a+7)$$
$$= -a \times a + -a \times 7$$
$$= -a^2 - 7a$$

b
$$-4b(2b-3)$$
$$= -4b \times 2b + -4b \times (-3)$$
$$= -8b^2 + 12b$$

44) Expand and simplify:

-a(a+1)=	-b(b+4)=	-c(5-c)=
-x(2x+4)=	-2x(1-x)=	-3y(y+2)=
-4a(5-a)=	-6b(3-2b)=	-3x(-x-2)=

Example 6 — Self Tutor

Expand and simplify:

a $3(x+5)+2(4-x)$ b $5(3-x)-2(x+1)$

a
$$3(x+5)+2(4-x)$$
$$= 3 \times x + 3 \times 5 + 2 \times 4 + 2 \times (-x)$$
$$= 3x + 15 + 8 - 2x$$
$$= 3x - 2x + 15 + 8$$
$$= x + 23$$

b
$$5(3-x)-2(x+1)$$
$$= 5 \times 3 + 5 \times (-x) + -2 \times x + -2 \times 1$$
$$= 15 - 5x - 2x - 2$$
$$= 15 - 2 - 5x - 2x$$
$$= 13 - 7x$$

45) Expand and simplify:

3(x+2)+2(x+3)=
4(x-2)+5(x-4)=
5(m+4)-3(m-2)=
5(x+3)+4(x+3)=
3(2x-7)+2(1-x)=
5(m-3)-2(m+1)=
3(x-1)-2(2x-4)=
7(1-x)-(3x+2)=
7(x+2)+3(2-4x)=

3(4-3x)-2(2x+1)=
-5(n-4)-3(2n-5)=
6(2y-1)+4(2-y)=
9(x+1)+3(2x-3)-15x=
11(2t-1)-3(5-3t)+4=

Example 7 Self Tutor

Expand and simplify:

a $4-2(x+3)$ b $8-3(2y-1)$

a
$$\begin{aligned} & 4-2(x+3) \\ &= 4+-2(x+3) \\ &= 4+-2\times x+-2\times 3 \\ &= 4-2x-6 \\ &= -2x-2 \end{aligned}$$

b
$$\begin{aligned} & 8-3(2y-1) \\ &= 8+-3(2y-1) \\ &= 8+-3\times 2y+-3\times(-1) \\ &= 8-6y+3 \\ &= 11-6y \end{aligned}$$

46) Expand and simplify:

3x-(2x+1)=
5-3(x+2)=
7-6(2x-3)=
11x-(2-x)=
6-5(1-2x)=
11-(3-2x)=
16-7(1-3x)=
x+6-3(4-x)=
8x+1-2(3-2x)=
7-(1-2x)=
2x-(8-7x)+3=
8-5(11-3x)=

Algebraic expressions. Numerical value *English*

To **evaluate** a mathematical expression we find its value for particular numerical substitutions of the variables or unknowns.
Evaluating a polynomial is to find its **numerical value** when the variables (x, y…) are replaced by any number.
If we substitute a negative value, we place in brackets. This helps us to evaluate the signs in the expression correctly. Although, to facilitate operations, we will place in brackets all the values that we substitute.

Example: consider the expression 5a + 3b
If a = -2 and b = 4 then 5a + 3b = 5(-2)+3·(4) = -10 +12= 2

Example*:* Evaluate $P(x) = 2x^3 + 5x - 3$ at x = 1
$P(1) = 2 \cdot (1)^3 + 5 \cdot (1) - 3 = 2 + 5 - 3 = 4$

Example*:* Evaluate $P(x) = 3x^6 + 2x^5 - 3x^4 - x^2 + 7x - 2$ at x=0
$P(0) = 3(0)^6 + 2(0)^5 - 3(0)^4 - (0)^2 + 7(0) - 2 = -2$
$Q(x,y)= -x^4y - x^2y + 7xy - 2$ at x = 1 and y = 2
$Q(1,2) = - (1)^4(2) - (1)^2(2) + 7(1)(2) - 2 = -2-2 + 14 - 2 = 8$

Operaciones algebraicas. Valor numérico *Spanish*

47) Calcula el valor numérico de las siguientes expresiones algebraicas para los valores de las variables indicados.

a) $3x+5y$
$en\ x=2\,,\ y=-1$

b) $x^2+2(3-y)$
$en\ x=2\,,\ y=-1$

c) $\dfrac{a(b+c)}{(c-a)a}$ *en* $a=3\,,\,b=4\,,\,c=5$

d) $x(x+1)(x-1)+3$
en $x=-1$

e) $x(x+1)(x-1)+3$
en $x=1$

f) $x(x+1)(x-1)+3$
en $x=3$

48) Dadas las siguientes expresiones algebraicas, tradúcelas a lenguaje ordinario y comprueba si se verifican las igualdades para los valores de las variables dados.

a) $4x-7=2$
para $x=3$

b) $3a-2b=7$
para $a=1\,,\,b=-2$

c) $\dfrac{p}{2}+\dfrac{p}{3}=q$
para $p=6\,,\,q=5$

49) Dadas las siguientes expresiones algebraicas, tradúcelas a lenguaje ordinario y comprueba si se verifican las igualdades para los valores de las variables dados. (*Notebook*)

a) $3(z-2)=6$
para $z=4$

b) $6m+2=\dfrac{16}{9-m}$
para $m=8$

c) $\dfrac{y}{3}+5=8$
para $y=9$

d) $2^{x+y}=32$
para $x=3\,,\,y=3$

e) $\dfrac{1}{s}+1=1'1$
para $s=10$

f) $p^2=13-q$
para $p=3\,,\,q=-4$

Expand and simplify. (*Notebook*)

a) $3(2-5a)-6(-a+1)$

b) $n^2+2(3-7n)+5n$

c) $-2z^2+z(z-1)-4z$

50) Expresa en lenguaje algebraico los siguientes enunciados. (*Notebook*)

a) En un aparcamiento hay ***c*** coches y ***m*** motos. ¿Cuántas ruedas hay?
b) En una granja hay ***g*** gallinas y ***c*** cerdos. ¿Cuántas cabezas hay?, ¿y cuántas patas? ¿Y cuántos ojos?
c) En un bolsillo tengo el doble de dinero que en el otro y en la mano 5€. ¿Cuánto dinero tengo en total?

51) Simplify the following algebraic expressions by combining like terms. (*Notebook*)

a) $-3a-3p+2p^3+7a-8p^3$

b) $1+x-13+7x^2+4x(1-2x)+1$

52) Representa las siguientes situaciones geométricas mediante un dibujo y una expresión algebraica. (*Notebook*)

a) The perimeter of a square of side ***x***
b) El área de un triángulo rectángulo de catetos ***a*** y ***b***
c) The area of a rectangle where one side is the double of the other.
d) El perímetro de un triángulo isósceles en el que el lado pequeño mide la tercera parte de cada lado grande.

53) Expresa en lenguaje algebraico los siguientes enunciados.

Adriana leyó el doble de libros que Alba, y Alba leyó 5 libros menos que Íker.
Luis tiene 3 veces la edad de Ramón. Eloy es 3 años mayor que Ramón.
Hay la décima parte de bolas rojas que de azules. Hay la mitad de bolas verdes que azules.

54) Expand and simplify. (*Notebook*)

a) $-7a(2-a)+15a(2-3a)$

b) $4m+5(m^2+8)-6m(8-2m)-3$

c) $1-t(8t-7)+7t^2+4t-2$

d) $10xy-y(8x-14)+2y+1$

55) Work out the numerical value of these algebraic expressions.

x	$3 \cdot (x-2)$
1	
-2	
0'5	

y	$\frac{y}{2}+1$
-8	
5	
0	

(a,b)	$2a-b^2$
(1,-1)	
(2,-4)	
(3,7)	

(x,y)	$x^2 \cdot y+4$
(2,-1)	
(-1,3)	
(-0,4)	

You may not use the calculator. Do the operations by hand.

56) Given the polynomials, (*Notebook*)

$p(x)= 2x^4 - 3x^3 - x^2 + 4x - 5$ $\quad$ $r(x)= x^2 - 3x + 2$

$q(x)= -3x^5 + 2x^4 - x^2 + 5x - 4$ $\quad$ $s(x)= 2x^2 - 3$

do the following operations:

a) $p(x)+q(x) - r(x)$
b) $3\cdot p(x) - 4\cdot q(x)$
c) $r(x)\cdot s(x) - p(x)$
d) $p(x)\cdot s(x)+2\cdot r(x)$
e) $-3\cdot q(x)+2\cdot p(x) - s(x)$
f) $s(x)\cdot q(x) - r(x)\cdot p(x)$

57) Calcula el valor numérico de los siguientes polinomios en el valor de la variable indicado en cada caso. (*Notebook*)

a) $p(x)= 3x^6 + 2x^5 - 3x^4 - x^2 + 7x - 2$ *en* $x = 0$

b) $r(x,y)= -x^4y - x^2y + 7xy - 2$ *en* $(x,y)= (1,2)$

c) $q(x)= -4x^2 + x + 5$ *en* $x = 2$

d) $s(x)= -2x^4 - 4x^2 - 2x - 3$ *en* $x = 2$

58) Calcula el valor de a para que el polinomio P(x)= 2x2 – ax +1 cumpla que P(2) = 5. (*Notebook*)

59) Given the polynomials, (*Notebook*)

$p(x)= 2x^5 - 3x^4 + 7x^3 - 2x^2 + 3x - 6$ $\quad$ $r(x)= 3x^2 - x + 1$

$q(x)= 3x^4 + 2x^3 + 5x^2 - 7x - 1$ $\quad$ $s(x)= 2x + 3$

do the following operations:

a) $[p(x) - q(x)] \cdot s(x)$
b) $[r(x) - q(x)] \cdot s(x)$
c) $[p(x) + q(x) - r(x)] \cdot s(x)$